Economics 113 — Prof. R. Starr
UCSD, Winter 2010 — Mr. Troy Kravitz

FINAL EXAMINATION

This exam is take-home, open-book, open-notes. You may consult any published source (cite your references). Other people are closed. The exam you turn in should be your own personal work. Do not discuss with classmates, friends, professors (except with Prof. Starr or Mr. Kravitz --- who promise to be clueless), until the examination is collected.

The completed exam may be submitted to Ms. Sydney Sprung at Sequoyah Hall 245; that office is open 7:30 - noon, 1:00 - 3:30. Submit by Friday, March 19, no later than 3:00 PM.

Do any five **(5)** *of the following seven problems. They count equally. Any paper submitting more than five problems will be counted on the lowest scoring five.*

Problem 14.18 *from Starr's "General Equilibrium Theory" Draft Second edition* Hint: The utility function is unusual. Most indifference curves we see bow in toward the origin. This utility function's indifference curves are rectangular and point out. They reflect tastes summarized as: "I like both x and y equally well, but I really do not like to consume both together."

Problem 14.19 *from Starr's "General Equilibrium Theory" Draft Second edition* Hint: Demand behavior in the neighborhood of a zero price is discontinuous here. How and why can that happen? What assumption of Theorem 14.1 is violated?

Problem 19.16 *from Starr's "General Equilibrium Theory" Draft Second edition*. Hint: Local non-satiation is the property that for any consumption bundle we give to household *i*, there is always another one nearby that *i* strictly prefers. Read through the proof of the First Fundamental Theorem of Welfare Economics (Theorem 19.1). Where is there an important step or equation that fails when the local non-satiation is not fulfilled? One of the following equations should fail. Which? Explain why.

$$x^i \succ_i w^{oi} \text{ implies } p^o \cdot x^i > p^o \cdot w^{oi} \qquad \textbf{(A)}$$

$$\sum_{i \in H} w^{oi} \leq \sum_{j \in F} y^{oj} + r \qquad \textbf{(B)}$$

$$p^o \cdot w^{oi} = M^i(p^o) = p^o \cdot r^i + \sum_j \alpha^{ij}(p^o \cdot y^{oj}) \qquad \textbf{(C)}$$

Problem 19.19 *from Starr's "General Equilibrium Theory" Draft Second edition* Hint: Household 1 is (relatively) poor and household 2 is (relatively) rich, but 2 really gets only limited enjoyment out of his riches because he can afford more than he actually enjoys. How does this situation fail to fulfill the assumptions of First Fundamental Theorem of Welfare Economics (Theorem 19.1)?

Problem 5. Consider a voting plan for a group of voters to choose the best one of ten possibilities: A, B, C, D, E, F, G, H, I, J. Each voter submits a ballot ranking the possibilities. The voting procedure then gives his first place choice a weight of 10; the second place choice is given a weight of 9; ...; the tenth place choice is given a weight of 1. For each possibility, the weighted votes of all the voters are then added up. The possibility achieving the highest total of weighted votes is declared the winner.

a. Evaluate the weighted voting procedure in terms of the Sen version of the Arrow axioms. Does the procedure fulfill: Pareto Principle? Independence of Irrelevant Alternatives? Non-Dictatorship? Unrestricted Domain? Explain.

b. Consider the following example to demonstrate whether voters find it advantageous to misstate their true preferences to influence the outcome. Let there be three voters with the following rankings. Topmost proposition is weighted 10, bottom is weighted 1:

Larry	Moe	Curly
A	D	G
B	E	H
C	F	I
D	G	J
E	H	A
F	I	B
G	J	C
H	A	D
I	B	E
J	C	F

Given this ranking G gets 21 points and looks like a winner (Prof. Starr can't do all these sums in his head --- he thinks that's right). Can Moe restate his preferences to make D a winner? How?

Problem 6. Consider a Robinson Crusoe economy with perfectly divisible labor. It is possible for Robinson to work ten (10) hours per day and leisure is not valued. There are two consumption goods, x and y. L^x is the amount of labor going to produce x each day, L^y is the labor going to produce y. The production functions are:

$x = (L^x)^2$, $y = (L^y)^2$, where the superscript “2” indicates a squared term and superscripts “x” and “y” merely indicate which good is being produced. The resource constraint is

$L^x + L^y = 10$.

(**part 1**) Robinson’s utility function is

$u(x, y) = x\cdot y$.

Robinson’s marginal rate of substitution

$MRS_{x,y} = \frac{y}{x} = \frac{u_x}{u_y}$, where the subscripts on u indicate partial derivatives.

A Pareto efficient allocation is (x, y) = (25, 25) , with $L^x = L^y = 5$. The obvious price vector to support this allocation is $(p_x, p_y) = (.5, .5)$ so that $MRS_{x,y} = \frac{p_x}{p_y} = 1$.

Demonstrate that there is no competitive equilibrium in this example. Explain why the Second Fundamental Theorem of Welfare Economics (Theorem 19.2 and Corollary 19.1) does not validly apply to this case. Hint: There is a scale economy in the production technology.

(part 2) Robinson’s utility function is

$u(x, y) = x + 6y$.

Robinson’s marginal rate of substitution

$MRS_{x,y} = \frac{u_x}{u_y} = \frac{1}{6}$

A Pareto efficient allocation is (x, y) = (0, 100) with $L^x=0$, $L^y= 10$. We can support this allocation as a competitive equilibrium with $(p_x, p_y) = \left(\frac{1}{6}, \frac{5}{6}\right)$.

Illustrate this situation with the usual Robinson Crusoe diagram: Production possibility set, optimizing indifference curve, maximal isoprofit line = budget line. You do not need to demonstrate that this is a competitive equilibrium. (Problem 6 continues next page).

We know from part 1 that this example does not fulfill Theorem 19.2. How then is it possible then to support the Pareto efficient allocation as a competitive equilibrium? Hint: Sufficient conditions versus necessary conditions.

Problem 7. Consider a population with the following preferences, denoted by >.

Household 1: A > B > C
Household 2: C > B > A
Household 3: B > A, B >C , A >C
Household 4: B > A, B > C, C > A
Household 5: C > B > A

We claim that this population --- or any odd numbered subset of this population --- has the property that majority voting on pairwise alternatives always results in transitive group choice, with the possibility of some ties. How is this possible? Is this a counterexample to the Arrow Possibility Theorem?